Mathematik Klasse 10b, AB 08 – Logarithmusrechenregeln – Lösung **13.12.2011**

Aufgabe 1: Schreibe den folgenden Term als Ausdruck einer einzelnen Logarithmusfunktion und vereinfache ihn so weit wie möglich.

a) $\log_2(72)-\log_2(9)=\log_2\left(\frac{72}{9}\right)=\log_2(8)=\mathbf{3}$

b) $\log_3\left(\frac{\log_b(a^9)}{\log_b(a)}\right)=\log_3\left(\log_a(a^9)\right)=\log_3(9)=\mathbf{2}$

c)
$$\log_a\left(\frac{4x^2-4y^2}{(2x-2y)^2}\right)+\log_a(x-y)=\log_a\left(\frac{(2x-2y)(2x+2y)}{(2x-2y)^2}\right)+\log_a(x-y)$$
$$=\log_a\left(\frac{(x+y)}{(x-y)}\right)+\log_a(x-y)=\log_a\left(\frac{(x+y)(x-y)}{(x-y)}\right)=\mathbf{\log_a(x+y)}$$

d)
$$\log_a(a^2b^3c^4)+\log_a\left(\frac{(a+b)}{(a^2-b^2)}\right)-\frac{\ln(b^3c^4)}{\ln(a)}$$
$$=\log_a(a^2)+\log_a(b^3c^4)+\log_a\left(\frac{\cancel{(a+b)}}{\cancel{(a+b)}(a-b)}\right)-\log_a(b^3c^4)$$
$$=2+\log_a\left(\frac{b^3c^4}{b^3c^4}\right)+\log_a\left(\frac{1}{a-b}\right)=2+\log_a(1)+\log_a\left(\frac{1}{a-b}\right)=2+\log_a\left(\frac{1}{a-b}\right)=2-\log_a(a-b)$$

e) $4\cdot\log_a(b^2-c^2)-\frac{lg(b+c)+lg(b-c)}{lg(a)} \quad = \quad 4\cdot\log_a(b^2-c^2)-\frac{lg((b+c)\cdot(b-c))}{lg(a)}$

$$=4\cdot\log_a(b^2-c^2)-\frac{lg(b^2-c^2)}{lg(a)} \qquad =4\cdot\log_a(b^2-c^2)-\log_a(b^2-c^2) \qquad =3\cdot\log_a(b^2-c^2)$$

f) $\log_a(a^2b^3c^4)-\log_a\left(\frac{(3a+3b)\cdot b^5}{(3a^2+6ab+3b^2)\cdot c^2}\right)+\frac{lg(b^2c^{-6})}{lg(a)}$
$$=2\cdot\log_a(a)+3\cdot\log_a(b)+4\cdot\log_a(c)-\log_a\left(\frac{3(a+b)b^5}{3(a+b)^2c^2}\right)+\log_a(b^2c^{-6})$$
$$=2+3\cdot\log_a(b)+4\cdot\log_a(c)-5\cdot\log_a(b)+\log_a(a+b)+2\cdot\log_a(c)+2\cdot\log_a(b)-6\cdot\log_a(c)$$
$$=2+\log_a(a+b)$$

g) $\log_a\left(\frac{x^2-2xy+y^2}{x-y}\right)-\log_a(x-y)=\log_a\left(\frac{(x-y)^2}{x-y}\right)-\log_a(x-y)=\log_a(x-y)-\log_a(x-y)=\mathbf{0}$

h) $2\cdot\log_a(a-b)-\frac{lg(a-b)}{lg(a)}=2\cdot\log_a(a-b)-\log_a(a-b)=\mathbf{\log_a(a-b)}$

i)
$$\log_a\left(\frac{\frac{\log_b(x^2y^2)}{\log_b(a)}+\frac{lg(x^3)}{lg(a)}+\frac{\frac{\ln(y^4)}{\ln(b)}}{\log_b(a)}}{\log_a(x^5y^6)}\right)=\log_a\left(\frac{\log_a(x^2y^2)+\log_a(x^3)+\frac{\log_b(y^4)}{\log_b(a)}}{\log_a(x^5y^6)}\right)$$

$$=\log_a\left(\frac{\log_a(x^2y^2)+\log_a(x^3)+\log_a(y^4)}{\log_a(x^5y^6)}\right)=\log_a\left(\frac{\log_a(x^5y^6)}{\log_a(x^5y^6)}\right)=\log_a(1)=0$$

Aufgabe 2: Bestimme x mit Hilfe von Äquivalenzumformungen so, dass es eine Lösung der folgenden Gleichung ist.

a) $-\log_a(x)=\log_a(8b^3c^4)-3\cdot\log_a(b)+2\cdot\log_a(c\,d\,e)+\log_a(\frac{1}{c^6})-\log_a(d^2e^2)$

$\Leftrightarrow -\log_a(x)=\log_a(8)+\log_a\left(\frac{b^3c^4c^2d^2e^2}{b^3c^6d^2e^2}\right)$

$\Leftrightarrow -\log_a(x)=\log_a(8)+\log_a(1)$

$\Leftrightarrow \log_a(\frac{1}{x})=\log_a(8)$

$\Leftrightarrow \frac{1}{x}=8$

$\Leftrightarrow x=\frac{1}{8}$

b) $-\log_a(\frac{1}{x})=\log_a(5b^4c^4)+2\cdot\log_a(b)+2\cdot\log_a\left(\frac{(d\,e)}{(b^3c^2)}\right)-\log_a(d^2e^2)$

$\Leftrightarrow -\log_a(\frac{1}{x})=\log_a(5)+4\cdot\log_a(b)+4\cdot\log_a(c)+2\cdot\log_a(b)+\log_a\left(\frac{(de)^2}{(b^3c^2)^2}\right)-2\cdot\log_a(d)-2\cdot\log_a(e)$

$\Leftrightarrow -\log_a(\frac{1}{x})=\log_a(5)+6\cdot\log_a(b)+4\cdot\log_a(c)+2\cdot\log_a(d)+2\cdot\log_a(e)-6\cdot\log_a(b)-4\cdot\log_a(c)$
$\qquad -2\cdot\log_a(d)-2\cdot\log_a(e)$

$\Leftrightarrow -\log_a(\frac{1}{x})=\log_a(5)$

$\Leftrightarrow \log_a(x)=\log_a(5)$

$\Leftrightarrow x=5$

c) $\log_b(x^2)\cdot\log_a(b)=\log_a\left[\left((a^2-b^2)+lg(10^{b^2})\right)^{\frac{\ln(a^2)}{\ln(a)}}\right]$

$\Leftrightarrow \log_b(x^2)\cdot\log_a(b)=\log_a\left[\left((a^2-b^2)+b^2\right)^{\log_a(a^2)}\right]$

$\Leftrightarrow \log_b(x^2)\cdot\log_a(b)=\log_a\left[\left(a^2\right)^2\right]$

$\Leftrightarrow \log_b(x^2)=\frac{\log_a(a^4)}{\log_a(b)}$

$\Leftrightarrow \log_b(x^2)=\log_b(a^4)$

$$\Leftrightarrow x^2 = a^4$$
$$\Leftrightarrow x_{1/2} = \pm a^2$$

d)

$$\log_k(x) = 2\cdot\log_k(4) + 3\cdot\log_k(3)$$
$$\Leftrightarrow \log_k(x) = \log_k\left(4^2\right) + \log_k\left(3^3\right)$$
$$\Leftrightarrow \log_k(x) = \log_k(16\cdot 27)$$
$$\Leftrightarrow \boldsymbol{x = 432}$$

e)

$$-\log_a(x) = 2\cdot\log_a\left(\frac{a^2 b^4}{c^3}\right) - \log_a(a^4) + \log_a(b^{-4}) + 3\cdot\log_a(c^2) + 2\cdot\log_a\left(\frac{a^2}{b^2}\right)$$
$$\Leftrightarrow -\log_a(x) = \log_a\left(\frac{a^4 \cancel{b^8} \cancel{c^6} \cancel{a^4}}{\cancel{c^6} \cancel{a^4} \cancel{b^4} \cancel{b^4}}\right)$$
$$\Leftrightarrow \log_a\left(x^{-1}\right) = \log_a\left(a^4\right)$$
$$\Leftrightarrow \frac{1}{x} = a^4$$
$$\Leftrightarrow \boldsymbol{x = \frac{1}{a^4}}$$

<u>Aufgabe 4:</u> Bestimme die Lösungsmenge der folgenden Exponentialgleichungen. Gib das Ergebnis mit zwei Stellen Genauigkeit hinter dem Komma an.

a)

$$3^{4x-6} = 81$$
$$\Leftrightarrow 3^{4x-6} = 3^4$$
$$\Leftrightarrow 4x - 6 = 4$$
$$\Leftrightarrow 4x = 10$$
$$\Leftrightarrow \boldsymbol{x = 2{,}5}$$

L = {2,5}

b)

$$2^x = 3\cdot 5^{x-1}$$
$$\Leftrightarrow 2^x = \frac{3}{5}\cdot 5^x$$
$$\Leftrightarrow lg\left(2^x\right) = lg\left(\frac{3}{5}\cdot 5^x\right)$$
$$\Leftrightarrow lg\left(2^x\right) = lg\left(\frac{3}{5}\right) + lg\left(5^x\right)$$
$$\Leftrightarrow x\cdot lg(2) = lg\left(\frac{3}{5}\right) + x\cdot\log(5)$$
$$\Leftrightarrow x\cdot\left(lg(2) - lg(5)\right) = lg(3) - lg(5)$$
$$\Leftrightarrow x = \frac{lg(3) - lg(5)}{lg(2) - lg(5)}$$
$$\Leftrightarrow \boldsymbol{x \approx 0{,}5575}$$

$$\boldsymbol{L = \{0{,}5575\}}$$

c)

$$4^{\left(x^2 - \frac{3}{16}\right)} = \left(\frac{1}{2}\right)^x$$
$$\Leftrightarrow lg\left(4^{\left(x^2 - \frac{3}{16}\right)}\right) = lg\left(\left(\frac{1}{2}\right)^x\right)$$
$$\Leftrightarrow \left(x^2 - \frac{3}{16}\right)\cdot lg(4) = x\cdot lg\left(\frac{1}{2}\right)$$
$$\Leftrightarrow x^2 - \frac{3}{16} = x\cdot\frac{lg\left(\frac{1}{2}\right)}{lg(4)}$$
$$\Leftrightarrow x^2 - \frac{3}{16} = -0{,}5\cdot x$$
$$\Leftrightarrow x^2 + 0{,}5\,x - \frac{3}{16} = 0$$
$$\Leftrightarrow x_{1/2} = -\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{3}{16}}$$
$$\Leftrightarrow$$
$$\Leftrightarrow x_{1/2} = -\frac{1}{4} \pm \frac{2}{4}$$
$$\Rightarrow \boldsymbol{x_1 = -\frac{3}{4};\ x_2 = \frac{1}{4}}$$
$$\boldsymbol{L = \{-3/4;\ 1/4\}}$$

d) $3{,}2^{3x-7}=9 \quad | \ \ln$

$\Leftrightarrow \ln\left(3{,}2^{3x-7}\right)=\ln(9)$

$\Leftrightarrow (3x-7)\ln(3{,}2)=\ln(9) \quad | \ :\ln(3{,}2)$

$\Leftrightarrow 3x-7=\frac{\ln(9)}{\ln(3{,}2)} \quad | \ +7$

$\Leftrightarrow 3x=\frac{\ln(9)}{\ln(3{,}2)}+7 \quad | \ :3$

$\Leftrightarrow x=\frac{\ln(9)}{3\cdot\ln(3{,}2)}+\frac{7}{3}$

$\Leftrightarrow \mathbf{x=2{,}96}$

$\mathbf{L=\{2{,}96\}}$

e) $\sqrt[3]{5^{7-x}}=5^{x-2} \quad | \ T$

$\Leftrightarrow \left(5^{7-x}\right)^{1/3}=5^{x-2} \quad | \ T$

$\Leftrightarrow 5^{1/3\cdot(7-x)}=5^{x-2} \quad | \ \log_5$

$\Leftrightarrow \frac{1}{3}(7-x)=x-2 \quad | \ \cdot 3$

$\Leftrightarrow 7-x=3x-6 \quad | \ -3x-7$

$\Leftrightarrow -4x=-13 \quad | \ :(-4)$

$\Leftrightarrow \mathbf{x=\frac{13}{4}}$

$\mathbf{L=\{\frac{13}{4}\}}$